A MECHANISM OF LIMITING THE DEVELOPMENT OF SELF-FOCUSING

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We consider the propagation of electromagnetic waves in a plasma. We shall assume that

$$E = E(\mathbf{r})\cos\omega t,\ \ \omega \gg \nu_e\delta;$$
$$l/\operatorname{grad} E/ \ll E,$$

where ν_e is the electron collision frequency, δ is the average fraction of energy lost by the electrons in one collision, and l is the mean free path of the electrons. Then in an inhomogeneous field the nonlinear thermal effects are stronger than the striction effects and the field expansion of the dielectric constant ε can be written in the following form [1]:

$$\varepsilon = \varepsilon_0 + i\varepsilon_1 + \varepsilon_2 \int \frac{\exp\left\{-\frac{|\mathbf{r}-\mathbf{r}'|\sqrt{3\delta}}{l}\right\}}{|\mathbf{r}-\mathbf{r}'|} |E(\mathbf{r}')|^2 d\mathbf{r}'.$$

If the variations of $|E(\mathbf{r})|^2$ over a distance $l_0 \sim l/\sqrt{\delta}$ (the dimension associated with thermal diffusivity) in the plasma are small, then we have

$$\varepsilon = \varepsilon_0 + i\varepsilon_1 + \varepsilon_2^1 |E(\mathbf{r})|^2, \tag{1}$$

which is valid also for a constant field [1, 2].

Hence, the stationary wave propagation in the plasma is described by the envelope equation

$$iv_0\frac{\partial\psi}{\partial z} = \Delta_\perp\psi + in_1\psi + n_2\int |\psi(\mathbf{r}', z)|^2 G(\mathbf{r}-\mathbf{r}')\, d\mathbf{r}'\psi, \tag{2}$$

where

$$G(\mathbf{r}-\mathbf{r}') = \int_{-\infty}^{+\infty} \frac{\exp\left\{-\frac{\sqrt{z^2+(\mathbf{r}-\mathbf{r}')^2}\sqrt{3\delta}}{l}\right\}}{\sqrt{z^2+(\mathbf{r}-\mathbf{r}')^2}}\, dz.$$

At the early stage of self-action, when the radial inhomogeneity of the beam is weak, according to (1), Eq. (2) is equivalent to

$$iv_0\frac{\partial\psi}{\partial z} = \Delta_\perp\psi + in_1\psi + n_2^1|\psi|^2\psi.$$

The term $in_1\psi$ is responsible for the attenuation of the wave; however, computations show that if the beam is intense, the process of self-focusing of the wave starts. For a well-developed self-focusing the term responsible for attenuation becomes small compared to the nonlinear term and the inhomogeneity of the field becomes important:

$$iv_0\frac{\partial\psi}{\partial z} = \Delta_\perp\psi + n_2\int |\psi(\mathbf{r}', z)|^2 G(\mathbf{r}-\mathbf{r}')\, d\mathbf{r}'\psi. \tag{3}$$

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Equation (3) has the following integrals of motion [3]:

$$I_1 = \int |\psi(\mathbf{r}, z)|^2 d\mathbf{r},$$
$$I_2 = -\int |\mathrm{grad}\,\psi(\mathbf{r}, z)|^2 d\mathbf{r} + \frac{1}{2}\int\int d\mathbf{r}d\mathbf{r}' G(\mathbf{r}-\mathbf{r}')|\psi(\mathbf{r}, z)|^2 |\psi(\mathbf{r}', z)|^2. \tag{4}$$

We shall obtain estimates showing that the growth of self-focusing ceases due to thermal diffusivity.

If A is the amplitude of the beam and L is its characteristic dimension, then from (4) we get

$$A^2L^2 \sim \mathrm{const}.$$

The term

$$\int |\mathrm{grad}\,\psi|^2 d\mathbf{r} > L^2A^2/L^2 \sim 1/L^2.$$

In the case of self-focusing, $L \to 0$, and for its existence it is necessary that the term

$$\int\int G(\mathbf{r}-\mathbf{r}')\ |\psi(\mathbf{r}, z)|^2|\psi(\mathbf{r}', z)|^2 d\mathbf{r}d\mathbf{r}'$$

grow no slower than $1/L^2$. For $r \lesssim l_0$,

$$G(\mathbf{r}-\mathbf{r}') \approx 2\ln(l_0/\sqrt{(\mathbf{r}-\mathbf{r}')^2}),$$

i.e., it is evident that

$$\int\int G(\mathbf{r}-\mathbf{r}')|\psi(\mathbf{r}, z)|^2|\psi(\mathbf{r}', z)|^2 d\mathbf{r}d\mathbf{r}' \sim \ln\frac{1}{L}.$$

It is thus shown that the growth of self-focusing will cease at scale sizes $L \sim l_0$, where l_0 is the dimension associated with thermal diffusivity in the plasma. It should be noted that for this result to be applicable the wavelength λ of the electromagnetic waves must satisfy the inequality

$$l \ll \lambda \ll l_0, \text{where } l_0 \sim 50l.$$

LITERATURE CITED

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